Indian Statistical Institute, Bangalore
B. Math (II), First semester 2016-2017
End-Semester Examination : Statistics (I).
Date: 15-11-2016 **Maximum Score 50** **Duration: 3 Hours**

1. Let $X_1, X_2, \cdots, X_n$ be a random sample from $N(\mu, \sigma^2)$ where μ is known to be integer valued and $\sigma^2 > 0$. Find the *maximum likelihood estimators (MLE)* for μ and σ^2. [12]

2. Consider the following *probability density function (pdf)*

$$\begin{aligned} f_{\mathrm{X}}(x|a,b,c) &= \frac{2(x-a)}{(b-a)(c-a)} \text{ for } a \le x \le c \\ &= \frac{2(b-x)}{(b-a)(b-c)} \text{ for } c < x \le b \\ &= 0 \qquad\qquad \text{otherwise,} \end{aligned} \tag{1}$$

   where $a < c < b$ and $a \in (-\infty, \infty)$. If $a < c < b$ are all known in (1), then how would you draw a random sample of size 15 using a **direct** method? Find the median of the distribution (1). [10 + 2]

3. Let $X_1, X_2, \cdots, X_n$ be a random sample from $N(0,1)$. Let us denote by R_j the rank of X_j, $1 \le j \le n$, defined as $\#\{i : X_i \le X_j\}$. Find $\rho_{\mathrm{R_1 R_n}}$. Let $Y_1 = \sum_{j=1}^{k} R_j$ and $Y_2 = \sum_{j=k+1}^{n} R_j$, $1 \le k < n$; find $\rho_{\mathrm{Y_1 Y_2}}$. [8 + 4 = 12]

4. Let $X_1, X_2, \cdots, X_n$ be a random sample from $Exp(\lambda)$. Derive the *likelihood ratio test* (LRT) for testing the hypotheses

$$H_0 : \lambda = \lambda_0 \; versus \; H_1 : \lambda \neq \lambda_0$$

   at level of significance α. Also obtain the p *value.* [10]

5. Tall cut-leaf tomatoes were crossed with dwarf potato-leaf tomatoes, and $n = 1611$ offsprings were classified by their phenotypes.

Phenotypes	Count
Tall cut-leaf	926
Tall potato-leaf	288
Dwarf cut-leaf	293
Dwarf potato-leaf	104

Genetic theory says that the four phenotypes should occur with relative frequencies in the ratio 9 : 3 : 3 : 1. Do the observed data support this theory? Take the level of significance α to be 0.05. Also report the *p-value.* [12]